AP Calculus BC Syllabus

Contents

Curricular Requirements

Label	Scoring Components	See Page(s)
CR1a	The course is structured around the enduring understandings within Big Idea 1: Limits.	5, 6, 8
CR1b	The course is structured around the enduring understandings within Big Idea 2: Derivatives.	6, 8
CR1c	The course is structured around the enduring understandings within Big Idea 3: Integrals and the Fundamental Theorem of Calculus.	7, 8
CR1d	The course is structured around the enduring understandings within Big Idea 4: Series.	8, 9
CR2a	The course provides opportunities for students to reason with definitions and theorems.	5, 7
CR2b	The course provides opportunities for students to connect concepts and processes.	6
CR2c	The course provides opportunities for students to implement algebraic/computational processes.	8
CR2d	The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.	5, 7
CR2e	The course provides opportunities for students to build notational fluency.	5, 6, 8
CR2f	The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.	5, 10
CR3a	Students have access to graphing calculators.	4
CR3b	Students have opportunities to use calculators to solve problems.	6
CR3c	Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.	5, 7
CR4	Students and teachers have access to a college-level calculus textbook.	3

AP®Calculus BC: Syllabus

Course Overview

This course offers a combination of assessment and instruction in an online environment containing but not limited to the areas of functions, functions and limits, differential calculus, and integral calculus. The course applies differential calculus to finding the slope of a curve, solving problems with related rates, calculating motion properties of moving particles, etc. It then applies integral calculus to finding the areas of irregular regions in a plane, finding volumes of rotation by various methods, and other scientific applications. Finally, the course explores analytic geometry, series, and convergence, as well as polynomial series and approximations.

The purpose of this course is to provide students with a deep understanding of the concepts of calculus in order to prepare them for the AP exam and for further college and university calculus courses. Because this class is presented in an online format, the pace and schedule varies from student to student, and no additional topics are presented past the exam time.

All the instruction is provided for the student, with feedback given for every exam, where students are required to give exact mathematical answers, often in sentence form, and describe, in detail, how solutions were arrived at, and the reasoning or theorems applied in the process. The teacher is also available at least five days a week for one-on-one support and help for the students, in addition to holding online collaborative lessons and offering discussion-based assessments to students.

Textbook

Larson, R. and Bruce H. Edwards. *Calculus of a Single Variable*. 10th ed. Boston: Brooks/Cole, 2006. **[CR4]**

Online Instruction

The Online Textbook powered by StudyForge technology consists of lessons which are delivered through multimedia flash videos with video and audio components, interactive applets and features built into every lesson, an accompanying workbook for taking guided, detailed notes for each lesson and doing a large variety of practice questions afterwards, and access to the Detailed Solutions for every practice question. In regards to the interactive nature of each lesson, this includes the use of applets. For example, the student varies the density of a slope field and moves the initial point around to a desired location, then changes the size of the solution curve through that initial value – or any combination of that. Another example would be investigating the mean value theorem for derivatives by testing the lower and upper limits on a continuous function to see whether a point, “c,” exists where the slope of the tangent line equals the slope of the secant line, etc. Interactive flash animations include rectilinear motion and varying the cut size of a cardboard to vary the size of the box created – all happening three-dimensionally in real time, rotating a cross-section of a square-based prism by dragging the mouse around so that the cross-section can be fully visualized, etc. Other features include built-in pauses during each lesson for the students to stop and reflect on the material, work on a question themselves before viewing the answer, or rewind the video and go through a section of the lesson again. (Note that at any time the student can pause the video, rewind, and even view at half speed.)

General lessons include investigation, application, theory, helpful tips, a heavy emphasis on intuitive understanding, and a very large number of worked out examples, in detail. The lessons are very comprehensive, and are hence broken down into three to four parts (typically) to aid in the learning process.

Graphing Calculators

Students are required to purchase and utilize personal graphing calculators while completing the course. **[CR3a]** The course recommends a TI-83 or a TI-84, and contains instructions for these calculators at various key points, as well as how to use the calculator under certain circumstances. Course instruction, practice free response items, formative, and summative assessments require that students use the calculator to interpret results and support conclusions including written explanations of results obtained by the calculator. All problems which specify the use of the calculator require students to interpret the result of the calculator and verbally, or through written communication, support conclusions rather than merely asking for the specific answer to the mathematical problem only.

Course Outline

Semester 1

Module One: Functions (suggested pace 2-3 weeks)

- Determine whether a relation is a function, as well as determine its domain and range. Sample activity: Students use the definition of a function to analyze a relation and use reason to defend their conclusion as to whether the relation is a function. **[CR2a]**
- Represent functions in a variety of forms including equations, tables, graphs, and verbal descriptions. Sample activity: Students are presented with a verbal description of a function and asked to produce a table, graph, and equation for the function. Students explain the connections between the notations in the different representations (numerical, graphical, analytical, and verbal). **[CR2d] [CR2e]**
- Use a graphing calculator to graph functions.
- Perform combinations of functions arithmetically or through composition, creating new functions; perform translations, reflections, and expansions/compressions on functions, and use technology such as a graphing calculator to experiment with such effects.
- Understand the properties of power functions, polynomials, rational functions, trigonometric functions; furthermore, use graphing calculators to explore the effects of changing any parameters of such functions on their corresponding graphs.

Module Two: Limits and Continuity (suggested pace 2-3 weeks)

- Evaluate the behavior of a function (in terms of limits) graphically (through sketching by hand and by using a graphing calculator), numerically, and algebraically. **[CR1a]**
- Understand the connection between vertical asymptotes and (infinite) limits, and use graphing calculators to support conclusions. **[CR1a]**
- Understand the connection between horizontal asymptotes and the "end behavior" of a function, and use a graphing calculator to help explore this concept.
- Use limits to describe the behavior of a function; use a graphing calculator to get numerical approximations for limits. **[CR1a]** Sample activity: Students use the definition of a limit to reason, analyze, and explain whether the limit exists on a function, and if so, determine the value of the limit. **[CR2a]**
- Use graphing calculators to examine functions near given values of x. Sample activity: Students graph functions on the calculator and use the graph and table features to explore whether these functions are continuous at a given point. **[CR2d] [CR3c]**
- Understand the connection between limits and continuity; use a function's continuity to evaluate its limit at a point; be able to determine when a function is continuous (or discontinuous), and understand how a graphing calculator can explore this further, as well as its limitations in that; identify the approximate roots of a function using the Intermediate-Value Theorem.
- Evaluate, support, and defend justifications both written and orally on continuity of piecewise functions presented via equations with missing components. Sample activities: In a collaborative online lesson, students present their justifications orally and engage in a discussion with other students. **[CR2f: oral]**. In an exam question, students provide their explanations in writing. **[CR2f: written]**
- Introduce various applications of limits, such as instantaneous and average rate of change, as

well as the motion of an object or particle: displacement, distance, and velocity. **[CR1a]**

Module Three: Differentiation (suggested pace 2-3 weeks)

- Understand the definition of the derivative as a local linear approximation, and what that implies, as well as differentiability and using graphs to explore tangent lines; understand the different notations for the derivative; explore the relationship between the graph of a function and its derivative; further explore the characteristics of the graphs of *f*, *f* ', and *f.*
- Find derivatives of polynomial functions. **[CR1b]**
- Use the product/quotient rules to find derivatives. **[CR1b]**
- Find derivatives of trigonometric functions. **[CR1b]**
- Use the chain rule (both of Newton and Leibniz's forms) to find the derivatives of composite functions. **[CR1b]**
- If possible, find the inverse of a function.
- Understand the properties of exponential and logarithmic functions; furthermore, use graphing calculators to explore the effects of changing any parameters of such functions on their corresponding graphs.
- Find derivatives of logarithmic functions. **[CR1b]**
- Find derivatives of exponential and inverse trigonometric functions. **[CR1b]**
- Use implicit differentiation to find the derivative/slope of a curve that is defined implicitly; use logarithmic differentiation to find derivatives. **[CR1b]**

Module Four: Applications of Derivatives (suggested pace 2-3 weeks)

- Determine the concavity of a function and discuss its implications on the shape of the curve; find critical points, local extrema, and points of inflection. Sample activity: A function is initially given in function notation and students are asked to analyze the function manually, and then use a graphing calculator to find local extrema of the given interval of the function. **[CR2e, CR3b]**
- Determine the intervals for where a function is increasing or decreasing - analytically, numerically and with a graphing calculator; sketch the curve of a function based upon information from its first and second derivatives – and vice versa. **[CR1b]**
- Determine the global or absolute extrema of a function on a closed interval, using both algebraic analytical techniques (with either the 1st or 2nd derivatives (or both)), as well as with the use of a graphing calculator. Sample activity: A function is initially given in function notation and students are asked to analyze the function manually, then use a graphing calculator to find all extrema of the given function. **[CR2e, CR3b]**
- Find the optimal values (maximums or minimums) in various application problems.
- Use derivatives to discuss the motion and rate of change of objects in terms of distance and displacement, velocity, speed and acceleration; use derivatives to discuss Rectilinear Motion (motion of a particle along a line). **[CR2b] [CR1b]**
- Use derivatives to solve related rates problems (being able to model how the rates of different quantities that depend upon the same parameter, such as time, interact).
- Use the Mean Value Theorem for Derivatives to make conclusions about a function on certain intervals (and points within those intervals), and explore these results via graphical methods; evaluate limits involving indeterminate forms using L'Hôpital's rule (for example: 0/0, ∞/∞, 0*∞, ∞-∞, 1∞, 00, and ∞0). **[CR1a]**
- Understand the definition of the derivative as a local linear approximation, and what that implies, as well as differentiability, and using graphs to explore tangent lines; understand the different notations for the derivative; explore the relationship between the graph of a function

and its derivative; further explore the characteristics of the graphs of f, f', and f''. Sample activity: Students are given information about f' and f'' and are asked to sketch f. **[CR2d]**

- Use local linear approximations (or differentials) to aid in approximation techniques

Module Five: Integration (suggested pace 2-3 weeks)

- Introduce integrals with Archimedes' Method of Exhaustion (numerical approximation) and how it leads to the natural use of the rectangle approximation method for the area under a curve; represent the area under a curve as a limit using sigma notation. **[CR1c]**
- Identify the definite integral as a limit of Riemann Sums; evaluate definite integrals by interpreting them geometrically; understand the differences or similarities, depending, between area (or "net signed area") and the definite integral, and explore this concept using a graphing calculator; further exploring Riemann Sums and accumulated change from a Rate of Change. **[CR3c] [CR1c]** Sample activity: Students are given a function and asked to evaluate the Riemann sum with six subintervals, graph the function and use it to reason and explain what the Riemann sum represents, and express the integral of the function as the limit of a Riemann sum. **[CR2a]**
- Use integrals to define functions, and explore that relationship; take the derivatives of integrals using the Fundamental Theorem of Calculus; use the Fundamental Theorem of Calculus to evaluate definite integrals. Sample activity: Students will use the Fundamental Theorem of Calculus to reason and evaluate $\int_0^3 (x^3 - 6x)dx$. Students will produce a graph of the function to explain the geometric meaning of the value of the integral and present their work to the instructor for discussion and feedback. **[CR2a] [CR1c]**
- Evaluate indefinite integrals to find the general antiderivatives of functions. **[CR1c]**
- Evaluate indefinite integrals by use of the method of substitution; evaluate definite integrals using the graphing calculator. **[CR1c]**
- Evaluate definite integrals using the method of substitution; evaluate definite integrals using the graphing calculator **[CR1c]**

Module Six: Applications of Integrals (suggested pace 2-3 weeks)

- Find the area between two curves using definite integrals, and explore this using technology. **[CR1c]**
- Use the method of discs/slicing/washers to find the volume of a solid of revolution.
- Explore and understand the Mean Value Theorem for Integrals; find the average value of a function; explore Rectilinear Motion with Integrals, as well as General Motion of Objects (Distance, Displacement, Velocity, Speed, and Acceleration). **[CR1c]**
- Evaluate constants of integration given an initial condition. **[CR1c]**

Semester 2

Module Seven: Differential Equations and More Riemann Sums (suggested pace 2-3 weeks)

- Evaluate constants of integration given an initial condition.
- Solve separable differential equations; model various applications using separable differential equations, with particular focus on the study of the equation y'= ky and exponential growth.
- Draw a slope field (or direction field) for a differential equation; be able to interpret a slope field when given it; be able to interpret the solution curve attached to an initial value.
- Use numerical approximation techniques to evaluate definite integrals, where appropriate,

including the area under a curve; use Riemann Sums (using left, right and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions through algebraic, graphical, and tabular representation (table of values); discuss error implications of different methods **[CR1c]** Sample activity: Students write a limit of a right-hand Riemann sum equal to the area of a region under a curve and use the Fundamental Theorem of Calculus to compute the area. **[CR2e]**

Module Eight: Supplemental Topics (suggested pace 2-3 weeks)

- Understand the definition of the derivative as a local linear approximation, and what that implies, as well as differentiability, and using graphs to explore tangent lines; understand the different notations for the derivative; explore the relationship between the graph of a function and its derivative; further exploring the characteristics of the graphs of f, f', and f''.
- Use numerical approximation techniques to evaluate definite integrals, where appropriate, including the area under a curve; use Riemann Sums (using left, right and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions through algebraic, graphical, and tabular representation (table of values); discuss error implications of different methods. Sample activity: Students are given data in a table and asked to use the trapezoidal sum to estimate the value of the integral. Students present their work in a collaborative environment where they discuss their work and receive feedback from their peers and instructor. **[CR2c]**
- Use integrals to define functions, and explore that relationship. **[CR1c]**
- Use integration by parts to compute antiderivatives and evaluate integrals, as well as recognize when it will be useful or not; use repeated integration by parts to evaluate integrals; include tabular method for repeated integration by parts. **[CR1c]**
- Use simple partial fraction decomposition (nonrepeating linear factors only) to compute antiderivatives and evaluate integrals, as well as recognize when it will be useful or not; be able to solve applications of partial fraction decomposition such as logistic differential equations, and use them in modeling. **[CR1c]**
- Identify integrals as improper and rewrite them as limits of definite integrals; determine whether an improper integral converges (to a finite limit) whether it diverges; compute definite integrals that are improper in applications, including the area under a curve or between curves and volume. **[CR1a] [CR1c]**

Module Nine: Analytic Geometry (suggested pace 2-3 weeks)

- Analyze planar curves given in parametric form; be able to compute derivatives of parametric functions and their applications such as finding tangent lines; find the arc length of a curve described in Cartesian form, as well as parametric form; convert between parametric form and other forms (such as rectangular/Cartesian form); graph curves in parametric form, including using technology to do so. **[CR1b]**
- Analyze planar curves given in polar form; be able to compute derivatives of polar functions and their applications such as finding tangent lines; find the area between polar curves; convert between polar form and other forms (such as rectangular/Cartesian form); graph curves in polar form, including using technology to do so.
- Analyze planar curves given in vector form; be able to compute derivatives of vector functions, including their application in rectilinear motion, and calculate related properties such as distance, displacement, velocity, acceleration, jerk, etc.

Module Ten: Series and Convergence (suggested pace 2-3 weeks)

- Review familiar series, including arithmetic and geometric series, and compute their partial sums, as well as their infinite sums if they exist (are convergent – particularly geometric series); find the general term of a series and identify whether it is arithmetic, geometric, or neither. **[CR1d]**
- Use graphs and technology to explore convergence and divergence; determine whether a sequence converges (to a finite value) or diverges; determine whether a series converges (to a finite value) or diverges; explore examples that involve the application of geometric series including repeating decimal expansion; view terms of series as areas of rectangles and their relationship to integrals. **[CR1d]**
- Test a series for convergence or divergence using the p-Series Test, Integral Test and Ratio Test. **[CR1d]**
- Test a series for convergence or divergence using the Ratio Test and the Alternating Series Test. **[CR1d]**
- Identify common series such as alternating series and the harmonic series; explore and compute the error bound of alternating series. **[CR1d]**

Module Eleven: Polynomial Series and Approximations (suggested pace 2-3 weeks)

- Calculate the n^{th} order Maclaurin polynomial approximation for a given function; be familiar with the Maclaurin series for the functions e^x, sin x, cos x, $\frac{1}{1-x}$; approximate a function with an appropriate Maclaurin Series. **[CR1d]**
- Calculate the n^{th} order Taylor polynomial approximation for a given function centered at x=a; approximate a function or value (such as ln3) with an appropriate Taylor Series; explore and compute the Lagrange error bound for Taylor polynomials; explore Taylor polynomial approximations to functions with graphical demonstration of convergence, including the use of appropriate technology to do so; use Taylor approximations in real-world applications such as modeling and calculating differential equations. **[CR1d]**
- Explore functions defined by power series; be able to find the power series of a given function by formally manipulating (and computing with) known power series, including the use of techniques such as substitution, differentiation, and integration. **[CR1d]**
- Calculate the interval and radius of convergence of a given power series; be familiar with the interval of convergence for the common power series (Maclaurin Series) for e^x, sin x, cos x, $\frac{1}{1-x}$.
- Use polynomial series to approximate irrational numbers. **[CR1d]**
- Use polynomial series to solve differential equations. **[CR1d]**
- Use polynomial series to do rectilinear motion. **[CR1d]**
- Use polynomial series to model real-world behavior. **[CR1d]**
- Use polynomial series to evaluate limits. **[CR1d]**

Module Twelve: Getting Ready for the Exam (suggested pace 2-3 weeks)

- All Previous Topics

Assessments

Students are assessed through a combination of quizzes, review assignments, module tests, teacher-student interaction, and the final exam.

- Students take an online, randomized multiple-choice quiz every lesson. These quizzes give them instant feedback as to their understanding of those lessons.
- The discussion-based assessments or collaborative lessons are such that the student receives detailed feedback about their methods, process and understanding, and they must reach a certain level of proficiency on the assignment (through resubmitting if necessary) before being allowed to take the chapter test for that unit. **[CR2f]**
- The module test contains two parts. The first part is a ten-question online randomized multiple choice test and the second part is a five-question essay exam where the students are required to show their work for each question, where they must demonstrate in detail their explanations, applications of theorems, mathematical proofs, etc., to explain exactly how they arrived at their answers. This written portion is worth significantly more marks than the actual answer itself. **[CR2f]**
- Students are required to both engage the teacher in verbal communication, both to receive any help if desired as well as be ready to defend their quiz or test answers and explain the solutions to their problems orally if requested, as well as answer essay-level questions and be able to discuss these questions thoroughly and accurately. **[CR2f]**